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Universal rules for fragmentation of land by humans

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Abstract The morphology of parcel patterns created by humans both in urban and rural areas is investigated. The parcel size distribution function, $f(a)$, provides a criterion, that enables unambiguous classification of each piece of land as city core, suburbs, or rural area. The morphology of the rural area corresponds to a scale-free structure and follows a power-law distribution $f(a) \sim a^{-n}$ of the parcel areas with the exponent $n \approx 1$. In suburbs, the area distribution follows the log-normal distribution. In the city core, $f(a)$ has an unimodal shape with an algebraically decaying tail, $n = 2$. Our study is based on data originating mainly from North America, the Hawaiian Islands, and Australia. For the regions analyzed, the characteristics of the parcel size distribution are universal and robust with respect to geographical, historical, and economical conditions accompanying development of a given area. The urbanization process can be described in terms of the changes of the morphology of the patterns of land fragmentation. In this formulation, the rural morphology, which can be thought as natural one because it exhibits a scale-free distribution of parcel sizes, is transformed into the artificial morphology developed in the city centers.

Key words Land parcels; Patterns; Morphology; Scaling laws; Urbanization process; Land fragmentation

Introduction

The ways in which human beings utilize land can be, in some cases, successfully modeled by natural processes known from biology or physics. For example, two-dimensional structures formed by the administrative division of land into districts (Weaire and Rivier 1984; Caër and Delannay 1993) or civil parishes (Pignol et al. 1993) have the same properties as natural cellular structures (Weaire and Rivier 1984) or the territory of fire ants (Adams 1998). Spatial distribution of cities on a plain (Glass and Tobler 1971) can be described using the classical model of liquids. Recent studies (Vicsek 1991; Makse et al. 1995, 1998; Zanette and Manrubia 1997; Manrubia et al. 1999; Batty and Longley 1994; Marsili and Zhang 1998) on human demography have shown that the ideas from statistical physics of aggregation and cluster growth can be successfully used to model both the fractal morphology of cities and the observed power laws governing the population frequency of cities (Zanette and Manrubia 1997; Marsili and Zhang 1998; Reed 2002; Blank and Solomon 2000). Prompted by the success of the geometrical approach, we study the morphology of a pattern created by humans in the process of fragmentation of land into the smallest units, i.e., parcels. We analyze the parcel patterns both in urban and rural areas located in different places around the globe and show that there exists an universal generic mechanism which people follow in their land processing.

Materials and Methods

In our analysis, we used the GIS (ESRI Inc. 2006) parcel data in the ESRI Shapefile format. We investigated data for North America, Australia, the Hawaiian Islands, and one European (Polish) city. The American data originate from eight counties and are available on the Internet (see Supplementary material for the addresses of the web sites). The areas we analyzed are located in different geographical regions, including lowland, mountain, and seaside terrains; populations of the cities varied from about one thousand to about two million inhabitants. In our studies a land parcel is the basic (smallest) spatial unit for cadaster and is assigned with a unique parcel number. The parcel is spatially defined with the parcel boundaries, and is represented by a polygon. The data contained all types of parcels, including both developed and undeveloped areas, green areas, public utilities, and industrial areas. Streets and roads were excluded from the analysis. The parcels were represented by sets of vertexes defining polygons. The centroids of the parcels were calculated as the centers of their weighted areas (i.e., centers of mass).

In our analysis, we employed the following procedure to determine the zones of the city: First, we selected a point being the “center of the city”. It was located in the geometric center of the Central Business District (CBD). In case of cities possessing multiple CBDs, the main CBD - also referred to as downtown - was chosen. Among all cities we studied only one – the city of Rockhampton, Australia – has two main CBDs of similar sizes. In this particular case, we selected the geometric center of the segment linking the centers of these two CBDs. Having selected the center of the city, we investigated the morphology of the parcel pattern of areas within the subsequent concentric rings. The width of the rings (the difference between the outer and the inner radius) varied in the range from 0.5 to 5 km. We checked that the areas within a selected ring exhibited the same morphology of the parcel pattern irrespectively of whether they belong to a city or not.

To investigate the morphology of the fragmentation pattern, we studied the distribution functions (histograms), $f(a)$, of the parcels areas, a. To prepare the distributions we applied the normalized logarithmic binning (Nlog) method (White et al. 2008), in which the values of the parcel areas are collected in logarithmically spaced bins, and the number of observations in each bin is normalized by the linear with of the bin. For the majority of the distributions we used 50 bins. In some cases, for the smallest sample sizes, 25 bins were used.

For the city cores, the tails of the distribution functions followed the power-law dependence

$$f(a) \sim a^{-n} \qquad (1)$$

In the case of rural areas, the distribution functions obeyed the above relation for all values of a except for the largest parcel areas. The exponents, n, were determined using two different methods. (i) First, the exponents were calculated as the slope of a linear model fitted through the log-log plot of the tail of the distribution (for all city cores), or the log-log plot of the distribution truncated at some large parcel size (for some of the rural areas). The distributions were prepared using the normalized logarithmic binning method. In each case, the data used to calculate the exponent was selected as set of points corresponding to the best compromise between having a large enough number of points and allowing for a good linear fit. (ii) Additionally, we calculated the values of the exponents using the maximum likelihood estimation (MLE) approach (White et al. 2008; Newman 2005), free from the arbitrariness of the binning of the data. Note that MLE is the best method of fitting parameters of distribution functions (White et al. 2008). The Nlog and MLE methods yielded very similar values of the power-law exponents.

At this point, note that most of the scaling work in landscape ecology and geography focuses on how different landscape (pattern) metrics respond to changing grain size or spatial extent (scale) of the area under investigation. Among a wide variety of different landscape metrics (Wu et al. 2002; Wu 2004) the commonly used are: the sum of areas of patches of a given type (Class Area), the number of the patches per unit area (Patch Density), average patch size, the percentage of the landscape covered by area of a given type, the percentage of the landscape comprised by the largest patch (Largest Patch Index), the measure of the shape complexity of patches (Landscape Shape Index), or the amount of edges of the patches relative to total landscape area (Edge Density). Wu et al. (Wu et al. 2002) analyzed 19 landscape metrics and examined how they respond to altering grain size and extent of the analyzed area. The authors cited found that only a few metrics exhibit consistent and robust scaling relations over a range of scales; the rest either exhibit staircase-like responses with changing scale or display erratic behavior with no consistent scaling relations. Recent studies (Wu 2004) showed also that the behavior of the landscape metrics obtained for selected land cover types (class levels) exhibit either simple scaling functions or unpredictable behavior. Similar behavior of the landscape metrics was also reported for deforestation (Frohn and Hao 2006) and forest fragmentation (Wickham et al. 2007) processes. In our studies we do not apply any of the landscape metrics mentioned above. The existence of the power laws for the parcel sizes distribution implies that the morphology of the land fragmentation pattern is self-similar and exhibits scaling properties. That is, if one changes the spatial extent of the landscape by some factor the shape of the distribution remains unchanged, except for some multiplicative constant. However, in contrast to the methods presented in the works cited, to obtain the scaling laws we do not analyze how the fragmentation pattern changes with grain size or spatial extent.

Results and Discussion

To investigate the morphology of the land fragmentation pattern, we studied the distribution functions, $f(a)$, of the parcels areas, a. Our studies revealed that the parcel patterns exhibit only three, well-defined morphological types. Each morphological type is unambiguously determined from the shape of the distribution function of the parcel sizes, $f(a)$. With respect to the morphology of the parcel pattern, one can classify each piece of land either as (i) highly urbanized core of a city, (ii) suburban area, or (iii) rural area. Here, the term “rural area” refers to non-urbanized land. It can include agricultural land as well as undeveloped land, green spaces, etc. These three classes of land, determined based on the function $f(a)$, are shown in Fig. 1. In the city core, $f(a)$ has a characteristic shape with a single peak located at the parcel size around 10^3 m^2, and the tail decays algebraically according to eq. (1) with the exponent $n = 2$. The core is surrounded by a ring of suburban area, characterized by the function $f(a)$ following the log-normal distribution. For rural areas, the distribution function obeys an inverse power-law scaling, $f(a) \sim a^{-n}$, with the exponent $n \approx 1$. Noticeably, the same behavior of the distribution function is observed even in the urban systems possessing a “reverse” structure, like that developed in the Hawaiian Islands shown in Fig. 1c. In this case, the core is the most outer part of the system and forms a thin ring along the cost of the island. The rural area is a circle located in the center, and is surrounded by a ring of suburbs. The features of the morphology of the parcel pattern shown in Fig. 1 are universal and do not depend on the topography or geographical location of land.

We found that in “regular” cities, the core (determined based on the parcel size distributions) occupies a relatively small part of the whole urban system and is enclosed in a circle centered in the middle of CBD. The radius of the core ranges from about 0.5 km for small towns to about 10 km for the largest cities. To determine the size of the core, we applied the following procedure: We investigated the distribution functions for parcels covered by concentric circles of increasing radii, R, with the center located in the middle of CBD. For small radii the function $f(a)$ exhibits the power-law tail with the exponent $n = 2$. When R exceeds some threshold value, R_{core}, the function $f(a)$ does not exhibit the power-law tail but – in the log-log plot – a concave tail. That is because the admixture of log-normally distributed parcel sizes from suburban area affects the tail of the distribution. The procedure described allowed fairly precise determination of the value of R_{core} - the size of the city core. It is illustrated in Fig. 2a for the city of Rockhampton (Australia, QLD). We see that for $R \leq R_{core} = 4$ km the tails of the distribution functions $f(a)$ follow the power-law scaling with $n = 2$. For bigger radii the shape of $f(a)$ changes noticeably and the distribution function exhibits features characteristic for both the city core and suburbs. Examples of the function $f(a)$ in the city cores for selected cities around the globe are shown in Fig. 2b. All distribution functions have the same, universal shape irrespective of the geographical location and the population of the city. The details concerning the populations, core sizes, and the values of the exponents for the 33 cities used in our analysis are given in Table 1. The majority of the cities are located in Australia (20) and USA (12); we investigated also one European city – the city of Krakow, which is an old Polish city established in the middle of 13th century. The fact that the city of Krakow has the same distribution of the parcel sizes as the Australian and American cities is remarkable. This fact strongly supports the hypothesis that there exists some robust mechanism which people follow around the globe in the urbanization process, and which is reflected in the morphology of the land fragmentation pattern.

We investigated the parcel size distributions in ten rural regions located in USA and Australia. The American regions are located in California and Maui Island in Hawaii (see Table 2 for details). The Australian regions are located in the state Queensland, and are marked on the map shown in Fig. 3. We found that all distribution functions follow an power-law dependence with the exponent $n = 1.1$. Examples of the functions $f(a)$ for four selected rural areas are plotted in Fig. 4a. The data concerning shapes, sizes, locations, and the values of the exponent n for each area are summarized in Table 2. The fact that in each case the distribution function obeys a power law suggests that people, in their activities that spontaneously contribute to land fragmentation, follow a scale-invariant process (Blank and Solomon 2000; Newman 2005; Salingaros and West 1999; Camacho and Sole 2001). In the log-log plot, the distributions are virtually straight lines apart from small parabolic bulges at their ends. These bulges reflect apparently a different type of land division process, which concerns only the largest parcels that are bigger than about 10^6 m^2. This large-scale process overlaps the spontaneous (i.e., scale-invariant) patterns of land fragmentation and was detected in the majority of the areas we analyzed. Such overlap can be attributed to an initial cadastral division. We observed that this large-scale land fragmentation is most pronounced in low-populated desert areas in Australia, where the natural human activity in land processing is fairly limited. This means that in rural areas, when urbanization processes are not present, land is gradually transformed into the natural scale-free structure as people continue to subdivide the parcels. Consequently, the initial large-scale structure vanishes.

In Fig. 4b the distribution functions for three regions located in Queensland, Australia are plotted. The degree of urbanization of land decreases with the distance from the center of Brisbane. The three regions represent different stages of urbanization. Region QLD(1) is the least urbanized (rural) area; Region QLD(4) comprises features of both rural and suburban areas; Region QLD(5), forming a ring around Brisbane, is a well-developed suburban area. We observed that regions located approximately at the same distance from the CBD of Brisbane – QLD(4) and QLD(7) – have very similar morphologies with overlapping distribution functions (Fig. 4c). Also, we found that the value of the average parcel size increases with the distance from the CBD, as illustrated in Fig. 4d.

The plots presented in Fig. 4 illustrate that the urbanization process is accompanied by the transformation of the power-law distribution characteristic of rural area (QLD(1)) into the log-normal distribution observed in suburban areas (regions QLD(4), ..., QLD(7)) surrounding the city of Brisbane. Also, based on Fig. 4, one can infer that if the land undergoes urbanization, the initial large scale patterns of fragmentation structure, represented by the parabolic bulge at the end of the distribution functions for QLD(1), grow at the expense of smaller parcels. Eventually, the parcel size distribution function takes the log-normal shape.

The analysis of the morphology of parcel pattern helps one to understand the possible mechanism of the land inclusion and fragmentation associated with the sprawl of urban systems on undeveloped (rural) areas. Namely, as a city grows its area is successively enlarged through the incorporation of adjacent suburban land. The newly included area undergoes then further urbanization. In the process, large parcels are successively sub-divided into smaller and smaller fragments. Eventually, this fragmentation process transforms the log-normal distribution function $f(a)$ into that presented in Fig. 2b, which is characteristic for the city core and signals that the urbanization process has reached its final stage. At the same time, the suburban area expands into surrounding rural areas. Since this process gives rise to the log-normal distribution of the parcel sizes, one expects that the inclusion and the transformation of the rural area into the suburban one is governed by a geometric Brownian motion (GBM). GBM is a stochastic multiplicative process (Reed 2002; Sornette and Cont 1997) in which the logarithm of randomly varying quantity, x, performs a Brownian motion. It leads to the log-normal distribution of the variable x. GBM accounts for both splits and merges of a parcel - two basic transformations that a parcel undergoes during the urbanization process. For this reason GBM is a natural candidate for the process that transformers the rural morphology into the suburban one.

To discuss possible mechanisms that would underlay the invariance in the distributions of the parcel sizes, we invoke recent results on human population ecology and note that distribution of the areas occupied by the world's nations was found (Buldyrev et al. 2003) to obey an inverse power law with the exponent $n = 0.93$. This value of the exponent is close to that of parcels sizes in rural areas, $n = 1.1$, reported in the present work. The process of land occupation that leads to the observed area distribution of nations can be modeled within the framework of random partitioning of the plane (Mekjian and Chase

1997; Buldyrev et al. 2003). In this process, the plane is successively subdivided by straight lines that are randomly oriented and positioned. Each line divides area available into two parts; the smaller part is selected and the larger one is further subdivided. This fragmentation mechanism results in an inverse power law distribution of the areas with the exponent $n \approx 1$. The fact that both the exponents – that of nations and that of parcel sizes - are close to unity supports the hypothesis that human activity that leads to the fragmentation of land in rural areas into parcels as well as the area distribution of nations around the globe is governed by the same generic mechanism. This mechanism is related to the geometrical properties of the partitioning of the plane, and does not depend on the biophysical and socio-political context. The value of the exponent $n = 2$ we found for the tails of the parcel sizes distributions in the city cores is common in human population ecology. The population distribution of cities obeys a power-law with $n = 2$ (the Zipf's law) (Zipf 1949, Makse et al. 1995, 1998; Zanette and Manrubia 1997; Gabaix and Ioannides 2004). Also, the area distribution of satellite images of cities, towns, and villages around large urban centers exhibits an inverse power-law with the exponent $n = 2$ (Makse et al. 1998). Marsili and Zhang (Marsili and Zhang 1998) proposed a simple stochastic model that reproduces the observed population distribution of cities. In this model, interacting individuals migrate and aggregate to form urban settlements. In short, the model comprises the tendency to live in a big city and the tendency to escape it due to all its negative impacts. A simple pairwise interaction between the individuals was assumed, accounting for complex social, economic, and cultural links between people living together. Importantly, the model discussed does not contain any ingredients depending on specific socio-economical conditions. The interactions among inhabitants occupying the same area might be also responsible for the spatial organization of a city, and, in particular, for the robust land fragmentation patterns formed in the city cores. The parcel sizes distribution would be a result of a compromise between the tendency of having maximum area of land in the center of a city and the price of land. Further studies are needed to identify the mechanisms that accounts for the invariances in land morphology we report in the present work.

Conclusions

To summarize, we have demonstrated that the morphology of the parcel pattern conforms to a criterion analogous to forms exhibited by natural phenomena; this criterion enables unambiguous classification of land as (i) the city core, (ii) suburbs, or (iii) rural area. The morphology of the rural area corresponds to a scale-free structure and is characterized by a reverse power-law distribution of the parcel areas with the exponent $n = 1.1$. In suburbs, the area distribution follows the log-normal distribution. In the city core, $f(a)$ has an unimodal shape with the peak located at around 10^3 m^2 and algebraically decaying tail, $f(a) \sim a^{-2}$. The urbanization process can be described in terms of the parcel size distributions as the transformation of the natural scale-invariant rural morphology into the artificial one formed in the centers of the cities. Our studies were based on the data originating mainly from USA and Australia, which have similar histories of colonization in terms of the mechanisms by which land was subdivided over time. We found, however, that the same features are exhibited by the city of Krakow, which is an old city (established in medieval times), located in Poland. Thus, we expect that the properties of the parcel area distribution function we discovered are generic and robust with respect to geographical, historical and economical conditions accompanying the development of a given urban or rural system.

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Tables

Table 1 Cities used in the analysis of the cores

City	Country	Population	Core (km)	*n*	
				Nlog	MLE
Blackall	AU	1,662	1.5	1.83 ± 0.07	2.00 ± 0.04
Brisbane	AU	958,504	10.0	2.07 ± 0.04	1.97 ± 0.01
Bundaberg	AU	45,873	2.5	1.99 ± 0.09	2.09 ± 0.03
Cairns	AU	125,327	7.0	1.71 ± 0.02	1.84 ± 0.01
Charters Towers	AU	8,846	1.5	2.15 ± 0.07	2.12 ± 0.06
Dalby	AU	10,215	2.0	1.78 ± 0.07	1.89 ± 0.03
Emerald	AU	13,523	2.0	1.82 ± 0.06	1.80 ± 0.03
Gladstone	AU	28,548	2.0	1.90 ± 0.08	1.83 ± 0.03
Gympie	AU	16,000	2.0	1.80 ± 0.08	2.08 ± 0.04
Innisfail	AU	8,394	1.5	1.71 ± 0.06	1.82 ± 0.05
Ipswich	AU	135,791	3.0	1.99 ± 0.04	2.10 ± 0.02
Mackay	AU	79,949	5.0	1.86 ± 0.06	1.74 ± 0.02
Maryborough	AU	25,635	2.0	2.06 ± 0.07	2.05 ± 0.05
Nanango	AU	4,500	2.0	1.74 ± 0.07	1.79 ± 0.03
Rockhampton	AU	59,755	4.0	1.80 ± 0.04	2.03 ± 0.02
Roma	AU	6,736	1.5	2.04 ± 0.09	2.13 ± 0.05
Stanthorpe	AU	10,592	1.5	2.05 ± 0.08	2.11 ± 0.04
Toowoomba	AU	94,189	5.0	1.94 ± 0.04	1.97 ± 0.01
Townsville	AU	98,075	10.0	1.65 ± 0.02	1.76 ± 0.01
Warwick	AU	21,564	2.0	1.97 ± 0.05	2.04 ± 0.03
Austin	USA	656,562	5.0	2.01 ± 0.03	2.09 ± 0.01
Bakersfield	USA	247,057	2.0	2.24 ± 0.08	2.23 ± 0.05
Fort Worth	USA	534,694	8.0	2.07 ± 0.03	2.10 ± 0.01
Georgetown	USA	28,339	2.0	1.82 ± 0.06	2.10 ± 0.05
Hawaii	USA	167,293	40.0-55.0[b]	1.86 ± 0.02	1.90 ± 0.01
Honolulu	USA	371,657	8.0-10.0[b]	1.97 ± 0.04	1.90 ± 0.02
Houston	USA	1,953,631	5.0	2.26 ± 0.03	2.10 ± 0.01
Kauai	USA	62,64[a]	23.0-24.0[b]	1.93 ± 0.09	1.95 ± 0.13
Maui	USA	139,884[a]	17.0-20.0[b]	1.91 ± 0.06	1.71 ± 0.02
Raleigh	USA	276,093	9.0	2.12 ± 0.03	2.10 ± 0.01
South Charleston	USA	1,850	0.5	2.10 ± 0.16	2.13 ± 0.06
Spokane	USA	195,629	5.0	2.10 ± 0.04	2.11 ± 0.01
Krakow	Poland	734,400	2.0	2.08 ± 0.04	2.00 ± 0.02

For all cities, the distribution function of the parcel areas, $f(a)$, decays algebraically with the exponent n, listed in the last column. The values of the exponents n were fitted using two different methods: linear regression based on the normalized logarithmic binning of the data (Nlog), and the maximum likelihood estimation (MLE).

[a] - Population of the whole island

[b] - The core forms a ring with the center located in the middle of the island. See also Table 2 for the details concerning location of the region.

Table 2 Regions used in the analysis of the rural areas

Country	Region	Shape	Size (km)	Centre (XY Coordinates)	*n* Nlog	*n* MLE
USA	Bakersfield	ring	30-35	(6258143,64, 2323873,89)[b]	1.01 ± 0.05	1.091±0.042
AU	Blackall	ring	30-50	(145.465323, -24.423086)[c]	1.00 ± 0.06	1.097±0.006
AU	Charters Towers	ring	40-50	(146.261218; -20.077380)[c]	0.97 ± 0.03	1.129±0.008
AU	QLD (1)[a]	circle	150	(142.918516, -16.651888)[c]	1.03 ± 0.03	1.118±0.007
AU	QLD (2)[a]	circle	150	(139.963044, -19.201577)[c]	0.95 ± 0.03	1.090±0.010
AU	QLD (8)[a]	circle	150	(152.439683, -27.021106)[c]	0.96 ± 0.05	1.102±0.003
USA	Maui	circle	6	(797804,05, 2295023,63)[d]	1.03 ± 0.03	1.146±0.017
USA	Hawaii	circle	34.5	(863338,72, 2175851,62)[d]	0.95 ± 0.04	1.130±0.008
USA	Kauai	circle	10.5	(448572,25, 2440014,39)[d]	0.99 ± 0.02	1.143±0.027
USA	Oahu	circle	2	(626651,02, 2359798,63)[d]	1.00 ± 0.02	1.232±0.055

For all regions, the distribution function of the parcel areas exhibits a power law behavior, $f(a) \sim a^{-n}$, with the exponents n, listed in the last column. The values of n were calculated using linear regression based on the normalized logarithmic binning of the data (Nlog), and the maximum likelihood estimation (MLE). Since the MLE method is considered as the most accurate for fitting the parameters of the distributions (White et al. 2008), we chose the value $n = 1.1$ as the estimate of the exponent n.

[a] See Fig. 3

Projected Coordinate Systems:

[b] NAD_1983_California_Zone_V

[c] GDA_1994_MGA_Zone_55

[d] NAD_1983_UTM_Zone_4N

Figures

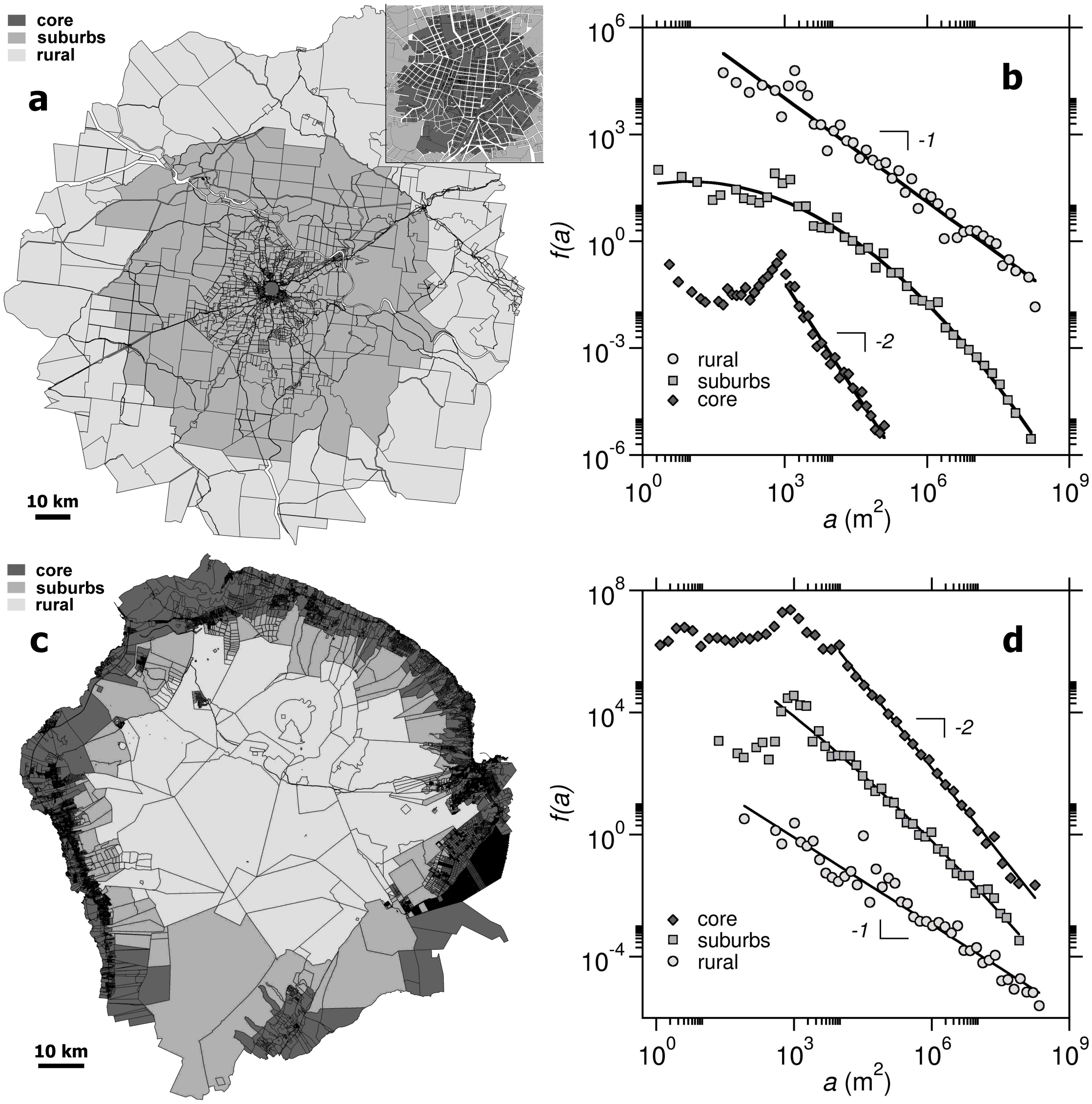

Fig. 1 (a) The city of Charters Towers (Australia, QLD) and its surroundings. The black lines are boundaries of the parcels. The three zones are determined according to the morphological class of the parcel pattern as: (i) the core, (ii) suburban area, and (iii) rural area. Insert: Details of the city core. **(b)** Log-log plot of the parcel areas distributions, $f(a)$, for the three zones shown in **(a)**. The core is characterized by the distribution function exhibiting the power-law tail with the exponent $n = 2$; in the rural area $f(a)$ follows a power-law with $n \approx 1$; in the suburban area $f(a)$ is described by the log-normal distribution, represented by a parabola in the log-log plot. **(c, d)** The three zones in case of Hawaii Island, being a *reverse* urban pattern: The rural area occupies a circle located in the center of the island. The rural area is surrounded by the ring of suburbs. The core of the city corresponds to the most outer ring. For the sake of clarity, the data sets have been mutually shifted in the vertical direction by multiplying the distribution functions by arbitrary numerical factors.

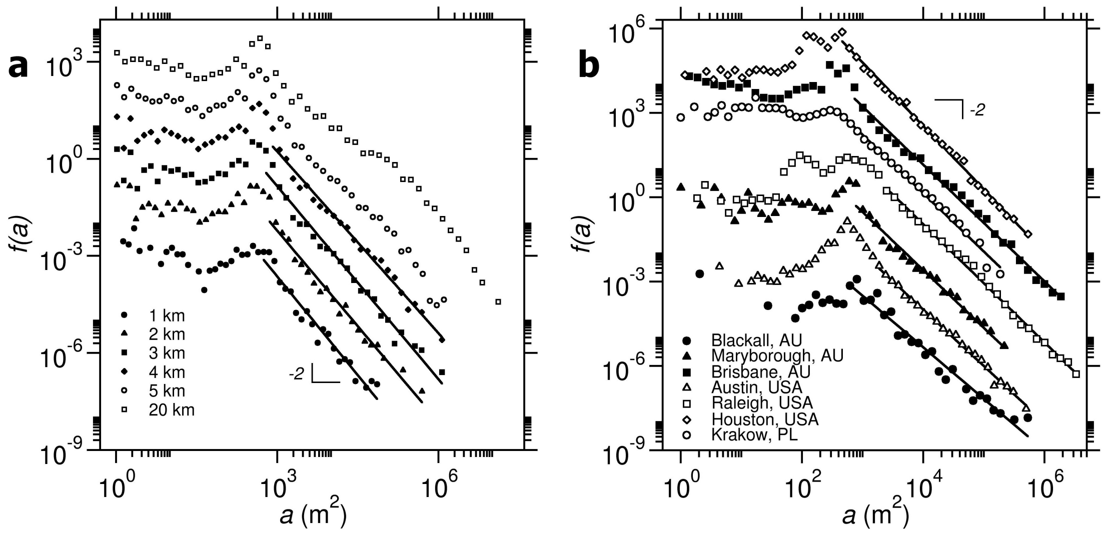

Fig. 2 (a) Log-log plots of the parcel size distribution functions, *f*(*a*), for Rockhampton (Australia, QLD). The radius of the city core (4 km) was determined based on *f*(*a*). As seen, within the core, *f*(*a*) have similar shapes with the characteristic power-law tails with $n = 2$. The functions *f*(*a*) for the parcels enclosed in the circles of the radius of 5 and 20 km have features characteristic of both the core and suburbs and do not exhibit the power-law tail; they display apparent concave tails without a straight line when drawn on a log-log scale. **(b)** Examples of *f*(*a*) for selected cities in Australia, North America, and Europe. All data sets have been mutually shifted in the vertical direction by multiplying the distribution functions by arbitrary numerical factors.

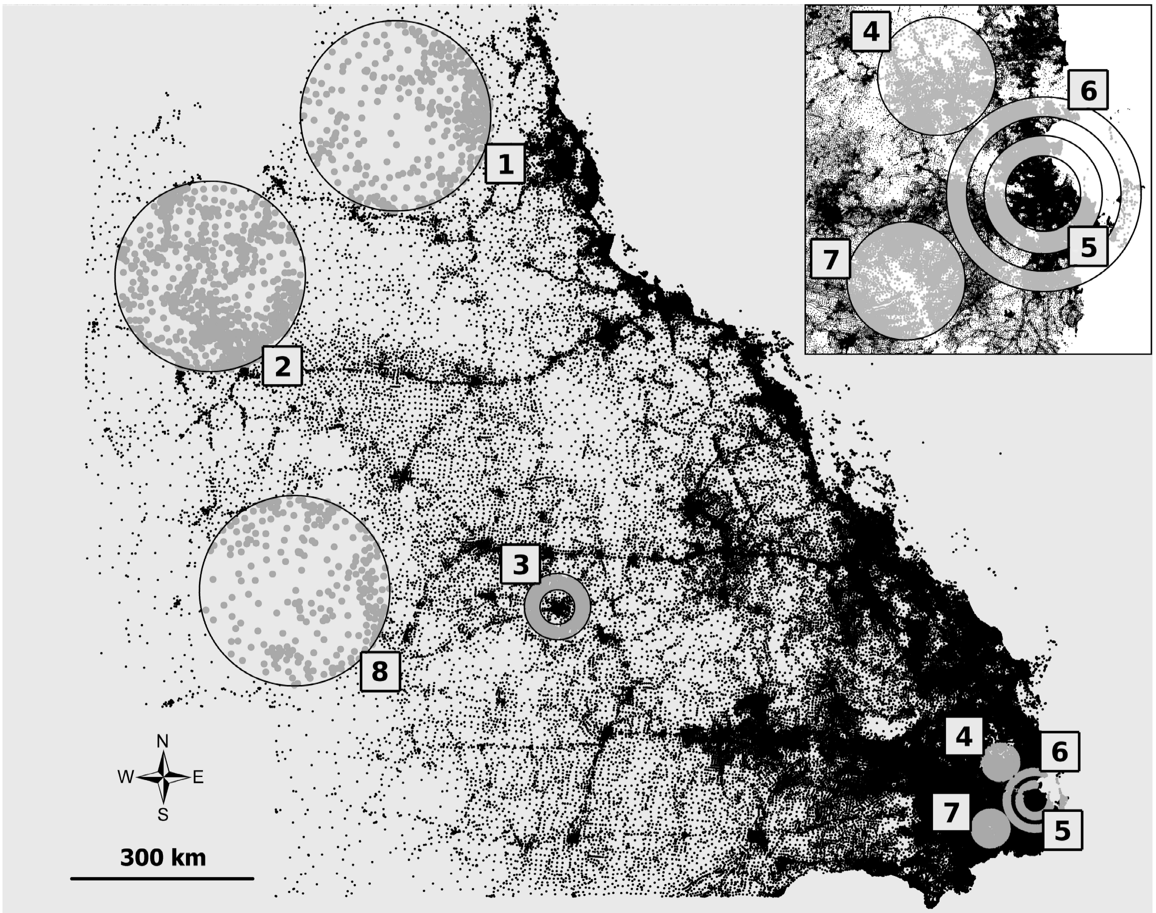

Fig. 3 Map of Queensland, Australia. The dots represent centroids of the parcels. The centroids in the regions analyzed are marked in gray and referred to as QLD(1), ..., QLD(8) in the text. Brisbane is located inside the ring "5".

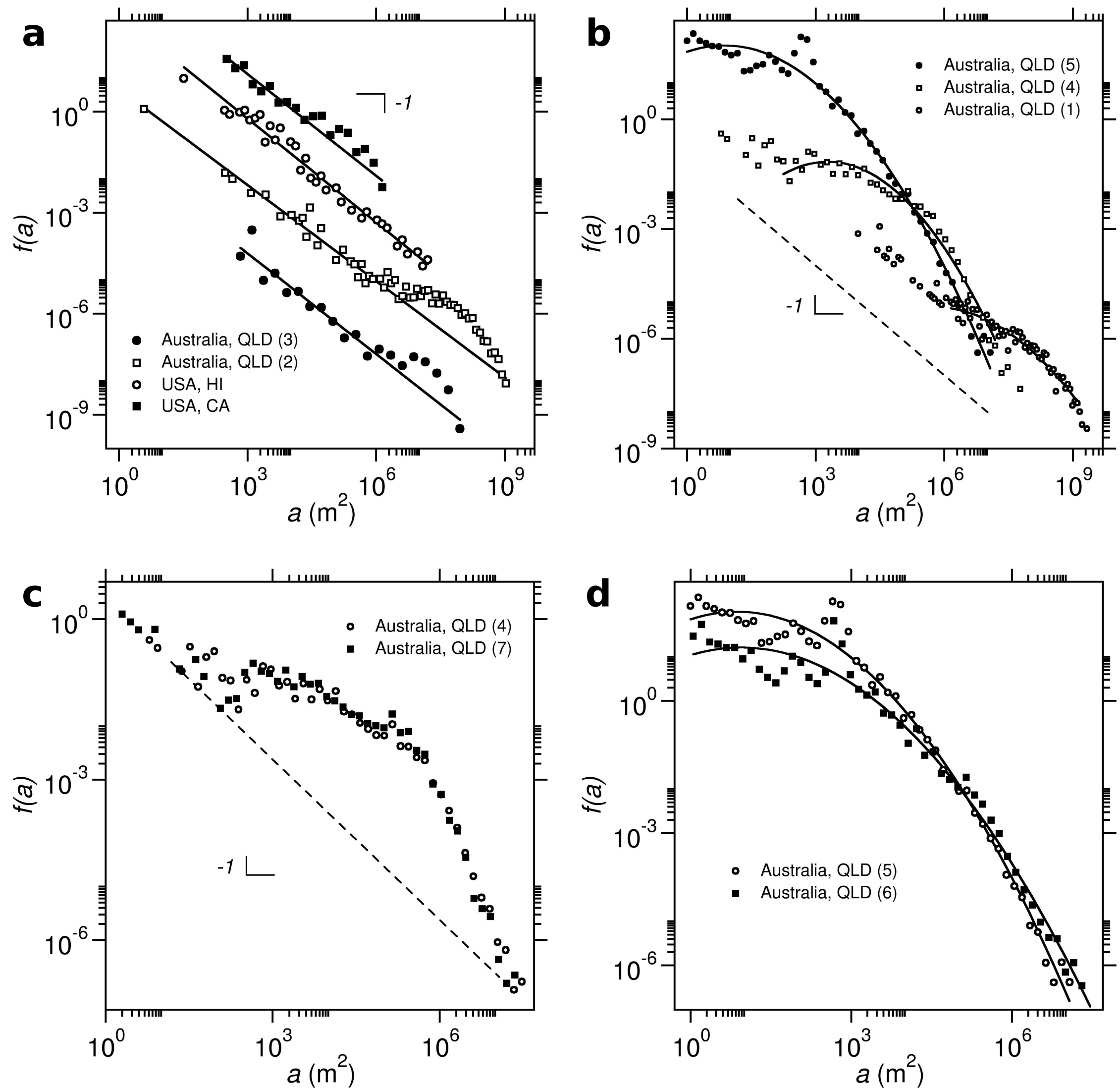

Fig. 4 (a) Log-log plots of the functions *f*(*a*) for four selected rural areas. The functions *f*(*a*) follow a power-law with the exponent $n = 1.1$. The small parabolic bulges at the ends of the distributions are due to initial large-scale processes of land division. **(b)** *f*(*a*) for three regions in Queensland, Australia representing different stages of urbanization. The scale-free distribution function characteristic of the rural area, QLD(1), transitions gradually into the log-normal distribution characteristic for suburban area, QLD(5). Function *f*(*a*) for QLD(4) exhibits a mixture of both rural and suburban features. **(c)** *f*(*a*) for the QLD(4) and QLD(7). These regions represent suburban areas and form circles with a radius of 30 km and are located at similar distances from the center of Brisbane. The two distributions collapse into each other. **(d)** *f*(*a*) for the regions QLD(5) and QLD(6) surrounding Brisbane. They represent rings of the sizes, respectively, 20–30 km and 40–50 km with their centers located in Brisbane. Both distributions represent the log-normal distributions. Region QLD(5), located closer to Brisbane, has smaller value of the average parcel area. The data sets in **(a)** and **(b)** have been mutually shifted in the vertical direction by multiplying the distribution functions by arbitrary numerical factors.

References

Adams, E. S. 1998. Territory size and shape in fire ants: A model based on neighborhood interactions. Ecology 79:1125-1134.

Batty, M., and Longley, P. 1994. Fractal Cities. Academic Press, San Diego.

Blank, A., and Solomon, S. 2000. Power laws in cities population, financial markets and Internet sites (scaling in systems with a variable number of components). Physica A 287:279-288.

Buldyrev, S. V., Dokholyan, N. V., Erramilli, S., Hong, M., Kim, J. Y., Malescio, G., and Stanley, H. E. 2003. Hierarchy in social organization. Physica A 330:653-659.

Camacho, J., and Sole, R.V. 2001. Scaling in ecological size spectra. Europhys. Lett. 55:774-780.

Caër, G. L., and Delannay, R. 1993. The administrative divisions of mainland France as 2D random cellular structures. J. Phys. I France 3:1777-1800.

ESRI (2006) ArcView 9.2, ESRI Inc. Redlands, CA, USA

Frohn, R. C., and Hao, Y. 2006. Landscape metric performance in analyzing two decades of deforestation in the Amazon Basin of Rondonia, Brazil. Remote Sensing of Environment 100:237-251.

Gabaix, X., and Ioannides, Y. M. 2004. The evolution of the city size distribution. In: Henderson, V., and Thisse, J. (Eds) Handbook of Urban and Regional Economics,Vol. 4, Elsevier, Amsterdam, pp 2341-2376.

Glass, L., and Tobler, W. R. 1971. Uniform distribution of objects in a homogeneous field - Cities on a plain. Nature 233:67-68.

Makse, H., Havlin, S., and Stanley, H. E. 1995. Modeling urban-growth patterns. Nature 377:608-612.

Makse, H. A., Andrade, J. S., Batty, M., Havlin, S., and Stanley, H. E. 1998. Modeling urban growth patterns with correlated percolation. Phys. Rev. E 28:7054-7062.

Manrubia, S. C., Zanette, D. H., and Sole, R. V. 1999. Transient dynamics and scaling phenomena in urban growth. Fractals 7:1-8.

Marsili, M., and Zhang, Y-Ch. 1998. Interacting individuals leading to Zipf's law. Phys. Rev. Lett. 80:2741-2744.

Mekjian, A. Z., and Chase, K. C., 1997. Disordered systems, power laws and random processes. Phys. Lett. A 229:340-346.

Newman, M. E. J. 2005. Power laws, Pareto distributions and Zipf's law. Cont. Phys. 46:323-351.

Pignol, V., Delannay, R., and Le Caër, G. 1993. Characterization of topological properties of 2D cellular structures by image analysis. Acta Stereol. 12:149-154.

Reed, W. J. 2002. On the rank-size distribution for human settlements. J. Reg. Sci. 41:1-17.

Salingaros, N. A., and West, B. J. 1999. A universal rule for the distribution of sizes. Env. Plan. B 26:909-923.

Sornette, D., and Cont, R. 1997. Convergent multiplicative processes repelled from zero: Power laws and truncated power laws. J. Phys. I France 7:431-444.

Vicsek, T. 1991. Fractal Growth Phenomena, 2nd edn. World Scientific, Singapore.

Weaire, D., and Rivier, N. 1984. Soap, cells and statistics - random patterns in 2 dimensions. Contemp. Phys. **25**:59-99.

White, E. P., Enquist, B. J., and Green, J. L. 2008. On estimating the exponent of power-law frequency distributions. Ecology 89:905-912.

Wickham, J. D., Riitters, K. H., Wade, T. G. and Coulston, J. W. 2007. Temporal change in forest fragmentation at multiple scales. Landscape Ecology, 22:481-489.

Wu, J. G., Shen, W. J., Sun, W. Z., and Tueller, P. T. 2002. Empirical patterns of the effects of changing scale on landscape metrics. Landscape Ecology 17:761-782.

Wu, J. G. 2004. Effects of changing scale on landscape pattern analysis: scaling relations. Landscape Ecology 19:125-138.

Zanette, D. H., and Manrubia, S. C. 1997. Role of intermittency in urban development: A model of large-scale city formation. Phys. Rev. Lett. 79:523-526.

Zipf, G. K. 1949. Human Behavior and the Principle of Least Effort. Addison-Wesley, Cambridge MA.