A hybrid MSV-MGARCH generalisation of the *t*-MGARCH model

Jacek Osiewalski[1], Anna Pajor[2]

Abstract

Hybrid MSV-MGARCH models, in particular the MSF-SBEKK specification, proved useful in multivariate modelling of returns on financial and commodity markets. Here we propose a natural hybrid generalisation of conditionally Student *t* MGARCH models. Our new hybrid EMSF-MGARCH specification is obtained by multiplying the MGARCH conditional covariance matrix H_t by a scalar random variable g_t, which comes from a latent process with auto-regression parameter φ that, for $\varphi = 0$, leads to an inverted gamma distribution for g_t and thus to the *t*-MGARCH case. If $\varphi \neq 0$, the latent variables g_t are dependent, so (in comparison to the *t*-MGARCH specification) in the new model of the observed time series we get an additional source of dependence and one more parameter. We apply the scalar BEKK specification as the basic MGARCH structure. Using the Bayesian approach, equipped with MCMC simulation techniques, we show how to estimate the new hybrid EMSF-SBEKK model. We present an empirical example that serves to illustrate the hybrid extension of the *t*-SBEKK model and its usefulness, as well as to compare it to the MSF-SBEKK case.

Keywords: *Bayesian econometrics, Gibbs sampling, time-varying volatility, multivariate GARCH processes, multivariate SV processes*

JEL Classification: C11, C32, C51

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1 Introduction

In volatility modelling of financial time series, hybrid MSV-MGARCH models were introduced by Osiewalski and Pajor (2007, 2009) and Osiewalski and Osiewalski (2016) in order to use relatively simple model structures that exploit advantages of both model classes: flexibility of the MSV class (where volatility is modelled by latent stochastic processes) and relative simplicity of the MGARCH class. In their first attempt, Osiewalski and Pajor (2007) used only one latent process and the DCC covariance structure proposed by Engle (2002). However, Osiewalski (2009) and Osiewalski and Pajor (2009) suggested an even simpler model, also based on one latent process, but with the scalar BEKK covariance structure. The parsimonious hybrid MSF-SBEKK specification has been recognized in the literature (see

[1] Corresponding author: Cracow University of Economics (Faculty of Management, Dept. of Econometrics and Operations Research); ul. Rakowicka 27, 31-510 Kraków, Poland, e-mail: eeosiewa@cyf-kr.edu.pl

[2] Cracow University of Economics (Faculty of Finance and Law, Dept. of Mathematics); ul. Rakowicka 27, 31-510 Kraków, Poland, e-mail: eopajor@cyf-kr.edu.pl

Teräsvirta, 2012; Amado and Teräsvirta, 2013; Carriero et al., 2016) and proved useful in multivariate modelling of returns on financial and commodity markets (see Pajor, 2010, 2014; Pajor and Osiewalski, 2012; Osiewalski and Osiewalski, 2012, 2013; Pajor and Wróblewska, 2017). Any MSF-MGARCH model amounts to using a conditionally normal MGARCH process and multiplying its conditional covariance matrix H_t by such positive random variable g_t that $\ln(g_t)$ follows a Gaussian AR(1) process with auto-regression parameter φ. If $\varphi = 0$, then such MSF-MGARCH case reduces to the MGARCH process with the conditional distribution being a continuous mixture of multivariate normal distributions with covariance matrices g_tH_t and g_t log-normally distributed.

In this paper we propose a natural hybrid extension of very popular MGARCH models with the Student t conditional distribution. Our new models are obtained by multiplying H_t by random variable g_t coming from such latent process (with auto-regression parameter φ) that, for $\varphi = 0$, g_t has an inverted gamma distribution and leads to the t-MGARCH specification, where the conditional distribution can be represented as a continuous mixture of multivariate normal distributions with covariance matrices g_tH_t and an inverted gamma distribution of g_t. If $\varphi \neq 0$, the latent variables g_t are dependent, so (in comparison to the t-MGARCH model) in the new model of the observed time series we get an additional source of dependence and one more parameter.

In the next section the general form of a MSV-MGARCH model as well as its special MSF-MGARCH structure are presented, and the EMSF-MGARCH class is formally defined. In Section 3 it is shown how to estimate the new hybrid EMSF-SBEKK model using the Bayesian approach and MCMC simulation techniques. In Section 4 an empirical example is presented; it serves to illustrate the hybrid extension of the t-SBEKK model and its validity, as well as to compare it briefly to the previous MSF-SBEKK specification.

2 Hybrid *n*-variate volatility specifications

Assume there are n assets. We denote by $r_t = (r_{t1} \ \dots \ r_{tn})$ n-variate observations on their logarithmic return (or growth) rates, and we model them using the basic VAR(1) framework:

$$r_t = \delta_0 + r_{t-1}\Delta + \varepsilon_t; \quad t=1, ..., T; \tag{1}$$

where T is the length of the observed time series. The hybrid MSV-MGARCH model class for the disturbance term ε_t is defined by the following equality:

$$\varepsilon_t = \zeta_t \, H_t^{1/2} G_t^{1/2}, \tag{2}$$

where: $\{\zeta_t\}$ is a strict n-variate white noise with unit covariance matrix, $\{\zeta_t\} \sim iiD^{(n)}(0, I_n)$;

H_t and G_t are square matrices of order n, symmetric and positive definite for each t; H_t is a non-constant function of the past of ε_t and corresponds to the conditional covariance matrix of some MGARCH specification; G_t is a non-constant function of a (scalar or vector) stochastic latent process $\{g_t\}$, which is non-trivial (i.e., constituted of variables g_t dependent over time); see Osiewalski and Osiewalski (2016). Under (1) and (2), the conditional distribution of r_t (given the past of r_t and the current latent variable g_t) is determined by the distribution of ζ_t; it has mean vector $\mu_t = \delta_0 + r_{t-1}\Delta$ and covariance matrix $\Sigma_t = G_t^{1/2}{}' H_t G_t^{1/2}$, which depends on both g_t and the past of r_t.

Building upon an idea presented by Osiewalski and Osiewalski (2016), we define a new subclass of the MSV-MGARCH model class. This subclass corresponds to the Gaussian white noise $\{\zeta_t\}$ and positive-valued scalar latent processes $\{g_t\}$ such that $G_t = g_t I_n$ and

$$\ln g_t = \varphi \ln g_{t-1} + \ln \gamma_t, \tag{3}$$

where $\zeta_t \perp \gamma_s$ for all $t, s \in \{1, ..., T\}$, $0< |\varphi| <1$ and $\{\gamma_t\}$ is a sequence of independent positive random variables with the same distribution belonging to a specific parametric family. The simplest MSV structure, called MSF and used by Osiewalski and Pajor (2009) to construct hybrid models with Gaussian $\{\zeta_t\}$, is based on the assumption that $\{\ln \gamma_t\}$ is a Gaussian white noise with unknown variance σ^2. In MSF-MGARCH hybrid models, (3) represents a two-parameter family of stationary and causal Gaussian AR(1) processes. Now we extend this basic case by considering other latent processes (3), corresponding to different parametric distribution classes of γ_t. We use the term "Extended MSF-MGARCH (EMSF-MGARCH) model" for all MSV-MGARCH models based on Gaussian $\{\zeta_t\}$ and distributions other than log-normal for γ_t in (3). In EMSF-MGARCH cases $\{\ln \gamma_t\}$ needs not be a white noise process.

In the MSF-MGARCH and EMSF-MGARCH cases, the conditional distribution of r_t (given its past and g_t) is Normal with mean μ_t and covariance matrix $\Sigma_t = g_t H_t$. In the MSF-MGARCH model, due to properties of Gaussian autoregressive processes, the marginal distribution of g_t is log-normal, so the distribution of r_t given its past (only) is the scale mixture of $N(\mu_t, g_t H_t)$ distributions with log-normal g_t. The mixing distribution depends on φ, and obviously remains log-normal for $\varphi=0$; this value leads to the MGARCH model with a specific ellipsoidal conditional distribution (the log-normal scale mixture of normal distributions). In the EMSF-MGARCH model class, the distribution of r_t given its past (only) is also the scale mixture of $N(\mu_t, g_t H_t)$ distributions, but we are not able to derive the marginal distribution of g_t, which obviously depends on φ. However, for $\varphi=0$ (the value excluded in the definition of our new hybrid models) $g_t=\gamma_t$, so the distribution of g_t is known by assumption. Since $\varphi=0$ corresponds to the MGARCH model with some ellipsoidal conditional distribution, i.e. the scale mixture of $N(\mu_t, g_tH_t)$ distributions with g_t distributed as γ_t, we may view any EMSF-MGARCH specification as a natural hybrid extension of the MGARCH model that is obtained for $\varphi=0$. In this paper we consider such EMSF-MGARCH extension of the popular MGARCH model with conditional Student *t* distribution. We focus on a particular, simple form of the MGARCH covariance matrix H_t, namely the scalar BEKK (SBEKK) form. It leads to the EMSF-SBEKK model based on an inverted gamma distribution of g_t; this is our hybrid extension of the *t*-SBEKK model.

3 Bayesian EMSF-SBEKK model and MCMC simulation of its posterior distribution

Assume that ε_t in (1) is conditionally Normal (given parameters and latent variables, jointly grouped in θ) with mean vector 0 and covariance matrix g_tH_t. The SBEKK form of H_t is as follows:

$$H_t = (1-\beta-\gamma)A + \beta\left(\varepsilon_{t-1}'\varepsilon_{t-1}\right) + \gamma H_{t-1}. \tag{4}$$

The univariate latent process $\{g_t\}$ fulfils (3) with γ_t^{-1} gamma distributed with mean 1 and variance $2/\nu$, i.e. $\{\gamma_t\} \sim iiIG(\nu/2, \nu/2)$.

In order to efficiently estimate our EMSF-SBEKK model, which is based on as many latent variables as the number of observations, we use the Bayesian approach equipped with MCMC simulation techniques. The Bayesian statistical model amounts to specifying the joint distribution of all observations, latent variables and "classical" parameters. The assumptions presented so far determine the conditional distribution of observations and latent variables given the parameters. Thus, it remains to formulate the marginal distribution of parameters (the prior or *a priori* distribution). We assume independence among almost all parameters and use the same prior distributions as Osiewalski and Pajor (2009) for the same parameters. The $n(n+1)$ elements of $\delta=(\delta_0 \text{ (vec } \Delta)')'$ are are assumed *a priori* independent of other parameters, with the $N(0, I_{n(n+1)})$ prior. Matrix A is a free symmetric positive definite matrix of order n, with an inverted Wishart prior distribution (for A^{-1}: Wishart prior distribution with mean I_n), and β and γ are free scalar parameters, jointly uniformly distributed over the unit simplex. As regards initial conditions for H_t, we take $H_0 = h_0 I_n$ and treat $h_0 > 0$ as an additional parameter (*a priori* Exponentially distributed with mean 1). φ has the uniform distribution over (-1, 1), for v we assume the Exponential distribution with mean $1/\lambda_v$ truncated to $(2, +\infty)$.

We can write the full Bayesian model as

$$p(r_1,...,r_T,g_1,...,g_T,\delta,A,\beta,\gamma,\varphi,v,h_0)=$$
$$=p(\delta)p(A)p(h_0)p(\beta,\gamma)p(\varphi)p(v)\prod_{t=1}^{T} f_N^n(r_t\,|\,\mu_t,g_tH_t)\times$$
$$\times\prod_{t=1}^{T}\frac{\left(\frac{v}{2}g_{t-1}^{\varphi}\right)^{\frac{v}{2}}}{\Gamma\left(\frac{v}{2}\right)}\left(\frac{1}{g_t}\right)^{\frac{v}{2}+1}e^{-\frac{\frac{v}{2}g_{t-1}^{\varphi}}{g_t}}. \tag{5}$$

The posterior density function, proportional to (5), is highly dimensional and non-standard. Thus Bayesian analysis is performed on the basis of a MCMC sample from the posterior distribution, which is obtained using Gibbs algorithm, i.e. the sequential sampling from the conditional distributions obtained from (5):

$$p(\delta\,|\,r_1,...,r_T,g_1,...,g_T,A,\beta,\gamma,\varphi,v,h_0)\propto p(\delta)\prod_{t=1}^{T} f_N^n(r_t\,|\,\mu_t,g_tH_t),$$

$$p(A\,|\,r_1,...,r_T,g_1,...,g_T,\delta,\beta,\gamma,\varphi,v,h_0)\propto p(A)\prod_{t=1}^{T} f_N^n(r_t\,|\,\mu_t,g_tH_t),$$

$$p(\beta,\gamma,h_0\,|\,r_1,...,r_T,g_1,...,g_T,\delta,A,\varphi,v)\propto p(\beta,\gamma)p(h_0)\prod_{t=1}^{T} f_N^n(r_t\,|\,\mu_t,g_tH_t),$$

$$p(\varphi\,|\,r_1,...,r_T,g_1,...,g_T,\delta,A,\beta,\gamma,v,h_0)\propto e^{\varphi\frac{v}{2}\sum_{t=1}^{T}\ln g_{t-1}}\times e^{-\frac{v}{2}\sum_{t=1}^{T}\frac{g_{t-1}^{\varphi}}{g_t}}\ I_{(-1,1)}(\varphi),$$

$$p(v\,|\,r_1,...,r_T,g_1,...,g_T,\delta,A,\beta,\gamma,\varphi,h_0)\propto\left(\frac{v}{2}\right)^{\frac{Tv}{2}}\Gamma\left(\frac{v}{2}\right)^{-T}e^{-\kappa v},$$

$$\text{where } \kappa=-\frac{1}{2}\sum_{t=1}^{T}\ln\frac{g_{t-1}^{\varphi}}{g_t}+\frac{1}{2}\sum_{t=1}^{T}\frac{g_{t-1}^{\varphi}}{g_t}+\lambda_v$$

$$p(g_t\,|\,r_1,...,r_T,g_1,...,g_{t-1},g_{t+1},...,g_T,\delta,A,\beta,\gamma,\varphi,v,h_0)\propto$$
$$\propto f_{IG}(g_t\,|\,n/2+(v/2)(1-\varphi),(1/2)(r_t-\mu_t)H_t^{-1}(r_t-\mu_t)'+(v/2)g_{t-1}^{\varphi})\,e^{-\frac{v}{2}\frac{g_t^{\varphi}}{g_{t+1}}};\quad t=1,...,T-1;$$

$$p(g_T\,|\,r_1,...,r_T,g_1,...,g_{T-1},\delta,A,\beta,\gamma,\varphi,v,h_0)$$
$$\propto f_{IG}(g_T\,|\,n/2+(v/2)(1-\varphi),(1/2)(r_T-\mu_T)H_T^{-1}(r_T-\mu_T)'+(v/2)g_{T-1}^{\varphi}).$$

Drawing from each conditional distribution above is done through Metropolis-Hastings steps.

4 An empirical example

In order to illustrate empirical validity of the EMSF–SBEKK model – in comparison to the pure GARCH, t-SBEKK specification – we use the same bivariate data sets as Osiewalski and Pajor (2009). The first data set consists of the official daily exchange rates of the National

Bank of Poland (NBP fixing rates) for the US dollar and German mark in the period 1.02.1996 – 31.12.2001. The length of the modelled time series of their daily growth rates (logarithmic return rates) is 1482. The second data set consists of the daily quotations of the main index of the Warsaw Stock Exchange (WIG) and the S&P500 index of NYSE. We model 1727 logarithmic returns from the period 8.01.1999–1.02.2006. In the case of exchange rates, both series are highly non-Normal and they are quite strongly positively correlated. The other data set shows smaller deviations from Normality and much weaker correlation.

Table 1. Posterior means (and standard deviations) of the parameters of the MSF–SBEKK and EMSF–SBEKK models for the exchange rates (T=1482).

parameter	**MSF–SBEKK**		**EMSF-SBEKK (λ_v = 1/10)**		**EMSF-SBEKK (λ_v = 1/30)**	
δ_{01}	0.044	(0.009)	0.040	(0.010)	0.040	(0.010)
δ_{02}	-0.005	(0.010)	-0.006	(0.010)	-0.006	(0.010)
δ_{11}	-0.020	(0.025)	-0.015	(0.025)	-0.014	(0.025)
δ_{12}	-0.012	(0.026)	-0.010	(0.026)	-0.009	(0.026)
δ_{21}	-0.012	(0.021)	-0.015	(0.021)	-0.015	(0.021)
δ_{22}	-0.040	(0.025)	-0.038	(0.025)	-0.039	(0.025)
a_{11}	0.153	(0.029)	0.079	(0.010)	0.079	(0.010)
a_{12}	-0.053	(0.018)	-0.024	(0.007)	-0.024	(0.007)
a_{22}	0.174	(0.034)	0.089	(0.012)	0.089	(0.012)
φ	0.411	(0.086)	**0.302**	**(0.096)**	**0.304**	**(0.096)**
σ^2 or v	0.540	(0.070)	**5.508**	**(0.573)**	**5.527**	**(0.575)**
β	0.084	(0.013)	0.068	(0.012)	0.068	(0.012)
γ	0.878	(0.015)	0.869	(0.016)	0.869	(0.016)
$\beta + \gamma$	0.962	(0.008)	0.937	(0.026)	0.937	(0.026)
h_0	0.053	(0.051)	0.038	(0.037)	0.039	(0.036)

In Tables 1 and 2 the posterior means and standard deviations of the MSF–SBEKK and EMSF–SBEKK parameters are presented for the exchange rates and stock indices data, respectively; the results for the MSF–SBEKK case are taken from Osiewalski and Pajor (2009). It is important to note that the posterior distribution of φ, the latent process auto-regression parameter, is further from zero in the MSF–SBEKK model for both data sets (especially for stock indices). It seems that the MSF–SBEKK model really needs the non-trivial Gaussian AR(1) latent process in order to describe the data, so that the case $\varphi = 0$, i.e. the SBEKK model with log-normal scale mixture as the conditional distribution, is clearly excluded. The question whether our EMSF–SBEKK model can be reduced to the *t*-SBEKK case cannot be answered so easily. For the exchange rates data, the posterior probability that $\varphi < 0$ is approximately 0.001 only and $\varphi = 0$ is included in the highest posterior density (HPD) interval of probability content as high as 0.998. Thus the *t*-SBEKK model is inadequate. But for the stock data, the posterior probability that $\varphi < 0$ is 0.061 for λ_v=1/10 and 0.045 for λ_v=1/30, and $\varphi = 0$ is included in the HPD interval of probability content 0.863 or 0.949, depending on the prior hyper-parameter λ_v. The *t*-SBEKK model cannot be rejected for the stock data and its empirical relevance is sensitive to the prior specification.

Table 2. Posterior means (and standard deviations) of the parameters of the MSF–SBEKK models for the stock indices (T=1727).

parameter	MSF–SBEKK		EMSF-SBEKK (λ_v = 1/10)		EMSF-SBEKK (λ_v = 1/30)	
δ_{01}	0.072	(0.026)	0.067	(0.026)	0.066	(0.026)
δ_{02}	0.028	(0.022)	0.026	(0.023)	0.026	(0.023)
δ_{11}	0.015	(0.024)	0.011	(0.024)	0.011	(0.024)
δ_{12}	0.012	(0.020)	0.009	(0.020)	0.010	(0.020)
δ_{21}	0.302	(0.027)	0.297	(0.026)	0.297	(0.026)
δ_{22}	-0.022	(0.026)	-0.023	(0.026)	-0.023	(0.025)
a_{11}	1.127	(0.267)	0.668	(0.141)	0.658	(0.141)
a_{12}	0.159	(0.104)	0.088	(0.070)	0.088	(0.071)
a_{22}	0.729	(0.176)	0.469	(0.097)	0.461	(0.098)
φ	0.872	(0.156)	**0.319**	**(0.220)**	**0.309**	**(0.209)**
σ^2 or v	0.036	(0.041)	**14.607**	**(5.054)**	**14.110**	**(3.798)**
β	0.021	(0.006)	0.032	(0.006)	0.032	(0.006)
γ	0.970	(0.007)	0.953	(0.008)	0.953	(0.008)
$\beta + \gamma$	0.991	(0.003)	0.985	(0.005)	0.985	(0.005)
h_0	2.881	(1.026)	2.277	(0.812)	2.252	(0.807)

The dependence of the marginal posterior distribution of φ on λ_v (the hyper-parameter of the prior of v), observed for the stock data, is intriguing. The parameters φ, v are independent *a priori* and only weakly dependent *a posteriori* in the case of exchange rates; their bivariate posterior distribution is almost the same for both values of λ_v (see Fig. 1). However, the joint posterior distribution of (φ, v) for the stock data, shown in Fig. 2, looks very different. It is bimodal, reveals a non-linear dependence between parameters and looks a bit different for λ_v=1/10 and for λ_v=1/30, with non-negligible second mode in the latter case. Note that the global mode corresponds to much lower values of the degrees of freedom and auto-regression parameter than the second mode. The larger φ (the stronger dependence in the latent process), the higher v (the thinner tail of the conditional distribution of the latent variable). Thus it is intuitive that λ_v=1/30, which gives high prior chances to thin tails (higher than λ_v=1/10 gives), may lead to the so high second mode. These subtle issues would require comparing different Bayesian models formally, which is not easy at all.

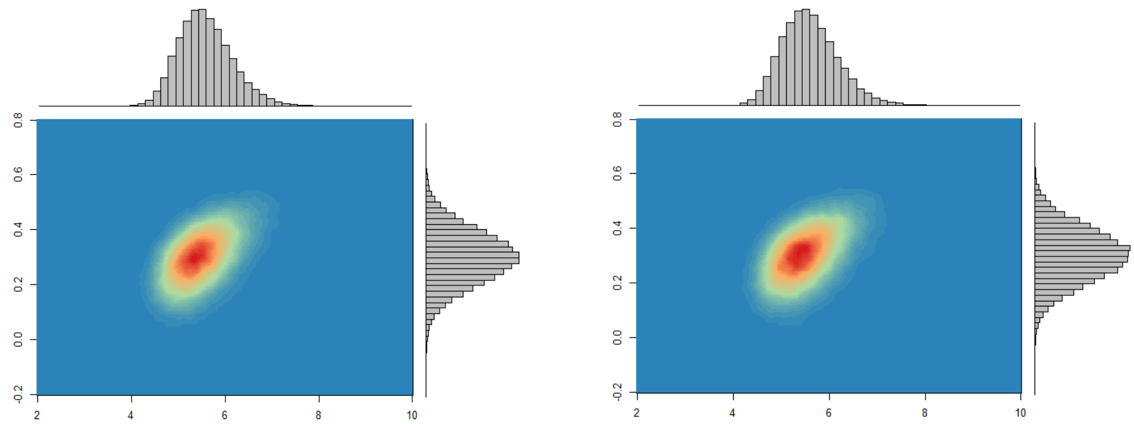

Fig. 1. The posterior distribution of (φ, v) in the EMSF-SBEKK model (exchange rates, T=1482; left panel: λ_v = 1/10; right panel: λ_v = 1/30).

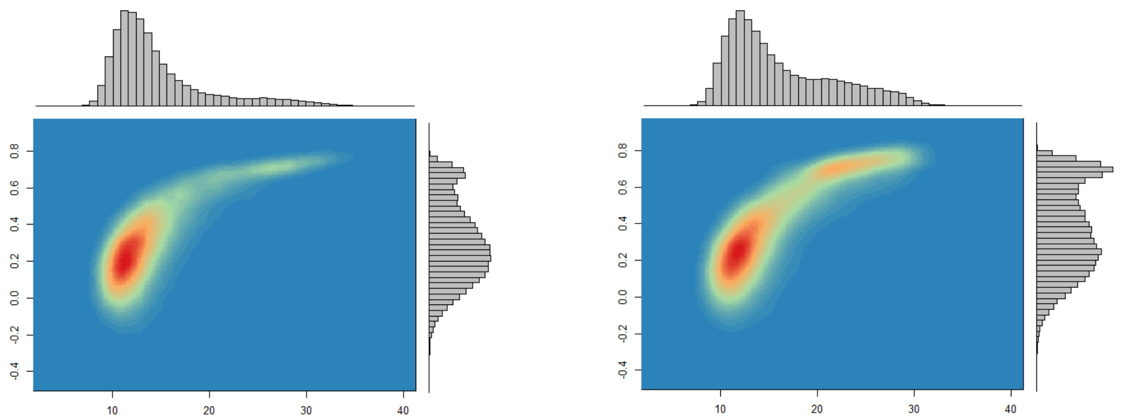

Fig. 2. The posterior distribution of (φ, v) in the EMSF-SBEKK model (stock indices, T=1727; left panel: λ_v = 1/10; right panel: λ_v = 1/30).

Note that the formal Bayesian model comparison (through Bayes factors and posterior odds) is computationally very difficult in our hybrid framework. The crucial issue is that of precisely calculating the numerical value (for the data at hand) of the marginal density of observations $p(r_1,..., r_T)$, which is the integral of the density (5) with respect to its all other arguments (i.e., latent variables and parameters). In order to approximate $p(r_1,..., r_T)$ within MCMC sampling from the posterior distribution, Osiewalski and Osiewalski (2013, 2016) used the harmonic mean estimator with a specific correction. Such approach does not have so good properties as the corrected arithmetic mean estimator (CAME) proposed by Pajor (2017). However, the use of CAME in dynamic models with latent processes is not numerically feasible yet, due to very high dimensions of the Monte Carlo simulation spaces. Thus, in this study we do not calculate the posterior model probabilities for the proposed EMSF-SBEKK model, its *t*-SBEKK limit case and the original MSF-SBEKK specification. The formal Bayesian comparison of alternative models is left for future research.

The aim of this paper was to show how to construct (and estimate within the Bayesian approach) a hybrid EMSF-MGARCH generalisation of the *t*-MGARCH model, focusing on the simple SBEKK structure. The Bayesian analysis of our EMSF-SBEKK model relies on Gibbs sampling with Metropolis-Hastings steps. Our illustrative empirical example suggests that the EMSF-MGARCH specification (that serves to generalise the *t*-MGARCH model) can relatively easily accommodate heavy tails – through latent process based on inverted gamma disturbances – in comparison to the MSF-MGARCH model, based on log-normal distribution and requiring larger values of the latent process auto-regression parameter φ.

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