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NEW HYBRID MODELS OF MULTIVARIATE VOLATILITY
(A BAYESIAN PERSPECTIVE)

1. INTRODUCTION

Most of multivariate volatility models used in financial econometrics either belong to the MGARCH or MSV (multivariate stochastic volatility or variance) classes or are based on copulas; see e.g. Bauwens, Laurent and Rombouts [1], Tsay [12]. These models are difficult to estimate; only a few of them could be practical tools for large portfolios. The ideal model should be both non-trivial and parsimonious. There are some candidates from the MGARCH class, e.g. the so-called "scalar BEKK" model and the Dynamic Conditional Correlation (DCC) structure of Engle [2]. In both cases one can use simple approximate methods (like variance targeting) in order to estimate the parameter vector of dimension growing with the portfolio size; the remaining parameters, which require more numerical effort, form a vector of fixed dimension (two) irrespective of the number of assets.

However, according to the Bayesian posterior odds criterion, MGARCH models do not explain the data as well as MSV specifications; see Osiewalski, Pajor and Pipień [6]. Latent AR(1) processes, used in the MSV class to describe volatility, are very efficient in dealing with outliers and, thus, in modelling tail behaviour. Since such modelling is crucial for any risk assessment, the MSV class should be kept under consideration in spite of the fact that reasonable MSV structures are too complicated to be practical in highly dimensional problems. Relatively easy, yet not trivial, multivariate volatility modelling is proposed by Osiewalski and Pajor [5], who define a hybrid model, based on Engle's DCC structure and the simplest MSV specification, the Stochastic Discount Factor (SDF) model. This paper is devoted to other hybrid models, of the SDF – scalar BEKK form, and to their approximate estimation that should result in feasible Bayesian analysis for large portfolios.

We follow the Bayesian approach to statistical modelling and inference, presented by e.g. O'Hagan [4]. The details for Bayesian MGARCH and MSV models are given by Osiewalski and Pipień [7], Pajor [9], [10] and Osiewalski, Pajor and Pipień [6].

Section 2 is devoted to simple MGARCH and MSV model structures, which are then combined in the new specification of section 3. Concluding remarks are grouped in section 4.

2. SIMPLEST BAYESIAN MODELS FROM THE MSV AND MGARCH CLASSES

Assume there are n assets. We denote by $r_t = (r_{t1} \ldots r_{tn})$ n-variate observations on their logarithimc return (or growth) rates, and we model them using the basic VAR(1) framework:

$$r_t = \delta_0 + r_{t-1} \Delta + \varepsilon_t; \; t = 1, \ldots, T, \ldots T + s, \tag{1}$$

where T denotes the length of the observed time series and s is the forecast horizon. The $n(n + 1)$ elements of $\delta = (\delta_0 \, (\text{vec } \Delta)')'$ are common parameters, which can be treated as *a priori* independent of all other (model-specific) parameters; we can assume for them some multivariate prior, e.g. standard Normal $N(0, I_{n(n+1)})$, truncated by the restriction that all eigenvalues of Δ lie inside the unit circle.

2.1. THE STOCHASTIC DISCOUNT FACTOR (SDF) MODEL

Assume that ε_t in (1) is conditionally Normal (given parameters and latent variables, grouped in θ) with mean vector 0 and covariance matrix Σ_t that depens on latent variables, i.e.

$$\varepsilon_t | \theta \sim N(0_{[1 \times n]}, \Sigma_t).$$

Thus, the corresponding conditional distribution of r_t (given its past and θ) is Normal with mean $\mu_t = \delta_0 + r_{t-1} \Delta$ and covariance matrix Σ_t. Competing n-variate MSV models are defined using different latent processes and different structures of Σ_t (symmetric and positive definite by construction). The simplest MSV specification uses only one latent process g_t to describe the dynamics of the whole conditional covariance matrix (see Jacquier, Polson and Rossi [3]; g_0 can be fixed, by assuming e.g. $g_0 = 1$):

$$\varepsilon_t = \zeta_t \sqrt{g_t}, \ln g_t = \phi \ln g_{t-1} + \sigma_g \eta_t, \zeta_t \sim iiN(0_{[1 \times n]}, A), \eta_t \sim iiN(0,1), \zeta_t \perp \eta_s \, (t, s \in Z).$$

The conditional covariance matrix of ε_t takes the very simple form $\Sigma_t = g_t A$, which leads to the invariable conditional correlation coefficient $\rho_{12,t} = \rho = a_{12} / \sqrt{a_{11} a_{22}}$; A is a free symmetric positive definite matrix consisting of $n(n + 1)/2$ distinct entries.

We can assume independence among parameters and use the same prior distributions as Pajor [10]; for ϕ: Normal with mean 0 and variance 100, truncated to (–1, 1), for A^{-1}: Wishart with mean I_n, for σ_g^{-2}: Exponential with mean 200.

In principle, the n-variate Bayesian VAR(1)-SDF model can be analysed using Gibbs sampling as a tool for simulating samples from the posterior distribution. This is due

to the Wishart or Normal forms of the full conditional distributions of A and δ, which make these steps of the Gibbs sampler easy even for large n. Other steps, numerically more demanding, do not depend on n; see Pajor [8], [9]. Despite its ease in practical applications, the SDF specification is too restrictive to be useful since it assumes the same dynamics for all entries of the conditional covariance matrix. This assumption seems too high a price to be paid for the ease of numerical implementation.

2.2. THE SCALAR N-BEKK(1,1) MODEL

Now we assume that the conditional distribution of ε_t (given past, ψ_{t-1}, and parameters) is n-variate Normal with mean vector 0 and covariance matrix H_t: $\varepsilon_t|\theta, \psi_{t-1} \sim N(0_{[1\times n]}, H_t)$. For their bivariate MGARCH models Osiewalski, Pajor and Pipień [6] assume the Student t distribution with location vector 0, inverse precision matrix H_t and $v > 2$ degrees of freedom, i.e. $\varepsilon_t|\theta, \psi_{t-1} \sim St(0_{[1\times n]}, H_t, v)$. The use of the Student t distribution instead of conditional Normality was a fully justified generalisation; see also Osiewalski and Pipień [7]. However, this more general specification adds numerical complexity, which is already high under n-variate Normality.

Particular n-variate GARCH models are defined by imposing different structures on H_t. Here we focus on a simple "scalar BEKK(1,1)" model as it can be easily estimated using approximate methods. It seems the simplest among multivariate structures allowing for dynamic conditional correlations. The scalar BEKK(1,1) case can be defined by

$$H_t = (1 - \beta - \gamma)A + \beta(\varepsilon_{t-1}'\varepsilon_{t-1}) + \gamma H_{t-1}, \tag{2}$$

where A is a free symmetric positive definite matrix of order n, with e.g. an inverted Wishart prior distribution, and β and γ are free scalar parameters, jointly uniformly distributed over the unit simplex. As regards initial conditions for H_t, we can either take $H_0 = h_0 I_n$ and treat $h_0 > 0$ as an additional parameter (*a priori* Exponentially distributed), or fix H_0.

Although the scalar N-BEKK(1,1) specification is so simple, its Bayesian analysis cannot rely on a fully automatic Gibbs algorithm. The full conditional posteriors of the VAR(1) parameters are no longer of the Normal form as δ also appears in H_t through the lagged VAR error terms. The full conditional posterior of A is not inverted Wishart as A is no longer the covariance matrix; it determines just one term of the sum defining the conditional covariance matrix in (2). The unwieldy form of the full conditional posterior for β and γ is not a problem since they are scalar parameters. However, δ and A are of high dimensions for large n and the use of Metropolis draws within the Gibbs steps (or using the Metropolis algorithm for all the parameters joinly) can be infeasible. Thus, any practical estimation tool must rely on some crude approximation when the analysed portfolio is very large. We can use OLS for the VAR(1) part and rely on variance targeting in order to estimate A; that is, A can be replaced by the empirical covariance matrix of the OLS residuals from the VAR (1) part. The Bayesian analysis for the two remaining scalar parameters and future return rates will then be based

on the conditional posterior and predictive distributions given the particular values of the highly dimensional parameters. We can use e.g. Monte Carlo with Importance Sampling for β and γ, and (for prediction purposes) direct Monte Carlo from the conditional Normal distributions of future return rates. Note that sampling of future return rates is unavoidable when the forecast horizon s is greater that 1, because then the joint s-period predictive distribution (given the parameters and observed data) is not Normal (although each one-period-ahead conditional distribution is).

3. HYBRID "SDF – SCALAR BEKK" MODELS

We specify the conditional distribution of the residual process ε_t by conditioning on its past (ψ_{t-1}), some univariate latent process (g_t) and the parameters. We assume in (1)

$$\varepsilon_t = \zeta_t \Omega_t^{1/2} \sqrt{g_t}, \ \ln g_t = \phi \ln g_{t-1} + \sigma_g \eta_t, \ (\zeta_t \eta_t)' \sim iiN(0_{[(n+1)\times 1]}, I_{n+1}),$$

that is, ε_t in (1) is conditionally Normal with mean vector 0 and covariance matrix $g_t \Omega_t$, where Ω_t is a time-varying square matrix of order n that preserves the scalar BEKK structure. Again, g_0 can be fixed as in the SDF model. We propose two particular forms of Ω_t.

In the type I hybrid model $\Omega_t = H_t$, so it follows (2) and does not depend on the latent process. Now the conditional variances are equal to $g_t \, h_{ii,t}$, that is they have a more general form than in either the SDF or $s$calar BEKK model, but the conditional correlation coefficient does not depend on g_t and thus is of the BEKK form. Note that this generalised structure does not lead to the MGARCH form of the process $\varepsilon_t^* = \varepsilon_t / \sqrt{g_t}$ as H_t depends on ε_{t-1}, not on ε_{t-1}^*.

In the type II hybrid model we assume the MGARCH structure for $\varepsilon_t^* = \varepsilon_t / \sqrt{g_t}$, i.e.

$$\Omega_t = H_t^*, \ H_t^* = (1 - \beta - \gamma)A + \beta(\varepsilon_{t-1}^* {}'\varepsilon_{t-1}^*) + \gamma H_{t-1}^*.$$

Now Ω_t depends on the whole past of the latent process, so do the conditional variances and correlation coefficients of ε_t. Modelling of time-varying conditional correlation is no longer as simple as in the scalar BEKK or type I hybrid models.

Both hybrid models seem useful as they combine important properties of their main structural components. The presence of one latent AR(1) process in the conditional covariance matrix should help in explaining outlying observations, and the dependence on the past data (through the BEKK structure of Ω_t) prevents the entries of the conditional covariance matrix $g_t \Omega_t$ from sharing the same dynamic pattern. Thus our models have time-varying conditional correlation without introducing more latent processes. In fact, the proposed hybrid models nest both basic structures. In the limiting case when $\sigma_g \to 0$ and $\phi = 0$ we are back in the BEKK model, while $\beta = 0$ and $\gamma = 0$ lead to the SDF case.

Assuming that the parameters of our hybrid specifications follow the same priors as in both special cases (SDF, BEKK), we can write the full Bayesian model as

$$p\left(r_1,...,r_{T+s},\ln g_1,...,\ln g_{T+s},\delta,A,\beta,\gamma,\varphi,\sigma_g^{-2}\right)$$
$$= p\left(r_{T+1},...,r_{T+s}\,\middle|\,r_1,...r_T,\ln g_1,...,\ln g_{T+s},\delta,A,\beta,\gamma,\varphi,\sigma_g^{-2}\right)$$
$$\times p\left(\ln g_{T+1},...,\ln g_{T+s}\,\middle|\,r_1,...,r_T,\ln g_1,...,\ln g_T,\delta,A,\beta,\gamma,\varphi,\sigma_g^{-2}\right)$$
$$\times p\left(r_1,...,r_T,\ln g_1,...,\ln g_T,\delta,A,\beta,\gamma,\varphi,\sigma_g^{-2}\right),$$

where

$$p\left(r_{T+1},...,r_{T+s}\,\middle|\,r_1,...,r_T,\ln g_1,...,\ln g_{T+s},\delta,A,\beta,\gamma,\varphi,\sigma_g^{-2}\right) = \prod_{t=T+1}^{T+s} f_N^n\left(r_t\,\middle|\,\mu_t,g_t\Omega_t\right),$$

$$p\left(\ln g_{T+1},...,\ln g_{T+s}\,\middle|\,r_1,...,r_T,\ln g_1,...,\ln g_T,\delta,A,\beta,\gamma,\varphi,\sigma_g^{-2}\right) = \prod_{t=T+1}^{T+s} f_N^1\left(\ln g_t\,\middle|\,\varphi\ln g_{t-1},\sigma_g^2\right),$$

$$p\left(r_1,...,r_T,\ln g_1,...,\ln g_T,\delta,A,\beta,\gamma,\varphi,\sigma_g^{-2}\right)$$
$$= p(\delta)p(A)p(\beta,\gamma)p\left(\varphi,\sigma_g^{-2}\right)\prod_{t=1}^{T} f_N^n\left(r_t\,\middle|\,\mu_t,g_t\Omega_t\right)f_N^1\left(\ln g_t\,\middle|\,\varphi\ln g_{t-1},\sigma_g^2\right).$$

(3)

The first two densities enable direct Monte Carlo simulation of future g_t and r_t; given all the parameters and (g_t, r_t) from the observation period ($t = 1, ..., T$), we successively draw $\ln(g_{T+j})$ and r_{T+j} ($j = 1, ..., s$) from their conditional Normal distributions. The last term, i.e. (3), is the joint density of the observed return rates, the T corresponding latent variables and all the parameters. The posterior density function, proportional to (3), is very complicated and highly dimensional. If n is large, the only hope to perform any Bayesian analysis is in the application of Gibbs sampling, which is based on full conditional distributions obtained from (3):

$$p\left(\delta\,\middle|\,r_1,...,r_T,\ln g_1,...,\ln g_T,A,\beta,\gamma,\varphi,\sigma_g^{-2}\right) \propto p(\delta)\prod_{t=1}^{T} f_N^n\left(r_t\,\middle|\,\mu_t,g_t\Omega_t\right),$$

$$p\left(A\,\middle|\,r_1,...,r_T,\ln g_1,...,\ln g_T,\delta,\beta,\gamma,\varphi,\sigma_g^{-2}\right) \propto p(A)\prod_{t=1}^{T} f_N^n\left(r_t\,\middle|\,\mu_t,g_t,\Omega_t\right),$$

$$p\left(\beta,\gamma\,\middle|\,r_1,...,r_T,\ln g_1,...,\ln g_T,\delta,A,\varphi,\sigma_g^{-2}\right) \propto p(\beta,\gamma)\prod_{t=1}^{T} f_N^n\left(r_t\,\middle|\,\mu_t,g_t\Omega_t\right),$$

$$p\left(\varphi,\sigma_g^{-2}\,\middle|\,r_1,...,r_T,\ln g_1,...,\ln g_T,\delta,A,\beta,\gamma\right) \propto p\left(\varphi,\sigma_g^{-2}\right)\prod_{t=1}^{T} f_N^1\left(\ln g_t\,\middle|\,\varphi\ln g_{t-1},\sigma_g^2\right),$$

$$p\left(\ln g_t\,\middle|\,r_1,...,r_T,\ln g_1,...,\ln g_{t-1},\ln g_{t+1},...,\ln g_T,\delta,A,\beta,\gamma,\varphi,\sigma_g^{-2}\right) \propto$$
$$f_N^1\left(\ln g_t\,\middle|\,\varphi\ln g_{t-1},\sigma_g^2\right)f_N^1\left(\ln g_{t+1}\,\middle|\,\varphi\ln g_t,\sigma_g^2\right)\prod_{j=t}^{T} f_N^n\left(r_j\,\middle|\,\mu_j,g_j\Omega_j\right); t = 1,...,T-1;$$

$$p\left(\ln g_T\,\middle|\,r_1,...,r_T,\ln g_1,...,\ln g_{T-1},\delta,A,\beta,\gamma,\varphi,\sigma_g^{-2}\right)$$
$$\propto f_N^1\left(\ln g_T\,\middle|\,\varphi\ln g_{T-1},\sigma_g^2\right)f_N^n\left(r_T\,\middle|\,\mu_T,g_T\Omega_T\right).$$

Again, as in the case of the pure scalar BEKK structure, we are not able to perform exact Bayesian analysis for a high enough portfolio dimension n. The conditional posteriors of δ and A, the parameters of dimension related to n, are non-standard. Using rejection sampling or the Metropolis-Hastings algorithm within these Gibbs steps would not be feasible for large n. However, we can still use the Gibbs sampling scheme (based on the full conditionals presented above) to propose some *ad hoc* approximate Bayesian approach. First of all, δ is of no particular interest and obtaining its posterior distribution is not important, so we can condition on its values resulting from the application of OLS to the VAR(1) system. We also need some quick approximation for A, but this will be proposed later. As regards β and γ, they can be sampled using the Metropolis-Hastings step within the Gibbs sampler.

Fortunately, the latent variables and parameters related to the SV component of the hybrid structure can be simulated in a similar way as in the pure SDF model, that is relatively easy. Under a Normal-Gamma prior for the pair $\left(\varphi, \sigma_g^{-2}\right)$, its bivariate conditional posterior is also of the Normal-Gamma form. And, most importantly, the univariate conditional posterior densities for $\ln(g_t)$ do not look too different from the pure SDF case. In fact, it pays to use the full conditional of $(g_t)^{-1}$ as it involves a Gamma kernel. In the type I model we obtain

$$p\left(g_t^{-1}\,\middle|\,r_1,...,r_T,g_1,...,g_{t-1},g_{t+1},...,g_T,\delta,A,\beta,\gamma,\varphi,\sigma_g^{-2}\right) \propto$$

$$f_G\left(g_t^{-1}\,\middle|\,\frac{n}{2},\frac{d_t}{2}\right)f_N^1\left(\ln g_t\,\middle|\,\varphi\ln g_{t-1},\sigma_g^2\right)f_N^1\left(\ln g_{t+1}\,\middle|\,\varphi\ln g_t,\sigma_g^2\right); t = 1,...,T-1;$$

$$p\left(g_T^{-1}\,\middle|\,r_1,...,r_T,g_1,...,g_{T-1},\delta,A,\beta,\gamma,\varphi,\sigma_g^{-2}\right) \propto f_G\left(g_T^{-1}\,\middle|\,\frac{n}{2},\frac{d_T}{2}\right)f_N^1\left(\ln g_T\,\middle|\,\varphi\ln g_{T-1},\sigma_g^2\right),$$

where $d_t = (r_t - \mu_t)\Omega_t^{-1}(r_t - \mu_t)'$. Since the log-Normal kernels can be approximated by some Gamma density, the Metropolis-Hastings steps have very efficient proposal density for $(g_t)^{-1}$; see Pajor [9]. In the type II model, the corresponding full conditional densities are more complicated due to the dependence of Ω_t on the past of g_t; we have for $t = 1, ..., T-1$:

$$p\left(g_t^{-1}\,\middle|\,r_1,...,r_T,g_1,...,g_{t-1},g_{t+1},...,g_T,\delta,A,\beta,\gamma,\varphi,\sigma_g^{-2}\right) \propto$$

$$f_G\left(g_t^{-1}\,\middle|\,\frac{n}{2},\frac{d_t}{2}\right)f_N^1\left(\ln g_t\,\middle|\,\varphi\ln g_{t-1},\sigma_g^2\right)f_N^1\left(\ln g_{t+1}\,\middle|\,\varphi\ln g_t,\sigma_g^2\right)\prod_{j=t+1}^{T}f_N^n\left(r_j\,\middle|\,\mu_j,\Omega_j\right).$$

The last product, which did not involve g_t and thus was omitted in the type I model, is now a complicated function of g_t. However, this term should be rather non-informative about g_t and, therefore, using the similar Gamma proposal density as in the type I model should be the basis for efficient Metropolis-Hastings steps within the Gibbs sampler.

Conditionally on some simple estimates of the VAR(1) parameters, we can perform Bayesian analysis of large portfolios, provided that A, which is a large symmetric matrix, can be either sampled or fixed. For the type I model we suggest conditioning on exactly the same estimate as in the case of the pure BEKK structure. In the case of

the type II model we can try a different estimate within each Gibbs pass. Due to the MGARCH property of the process $\varepsilon_t^* = \varepsilon_t / \sqrt{g_t}$, we can use variance targetting and (at each Gibbs pass) estimate A by the empirical covariance matrix of $\varepsilon_t^*, t = 1, \ldots, T$. Since this matrix is different at each Gibbs pass, we finally obtain a "sample" of reasonable estimates of A. This should give some idea about the location of the important part of the posterior of this matrix parameter, and thus should be better than just fixing it. However, if there are any convergence problems, we can use OLS residuals to fix A (as in the case of the pure BEKK or type I hybrid model).

4. CONCLUDING REMARKS

Any statistical analysis of a large portfolio requires compromises. It needs relatively simple (but non-trivial) multivariate volatility models. In addition, any Bayesian analysis has to partly rely on conditioning on particular parameters (using non-Bayesian estimates). However, there are promising classes of hybrid MGARCH-MSV structures, where one can use Bayesian inference on the most important model components: latent variables, deeper parameters and future returns. Note that our new specifications formally belong to the MSV class due to the presence of the latent AR(1) process describing multivariate volatility. But we use the term "hybrid models" in order to stress their difference from traditional "pure" MSV conditional covariance structures, which do not depend on past observations.

Before using the proposed approximate approach for very large portfolios, it should be compared to the full and exact Bayesian analysis in low dimensional situations. Also, the usefulness of our Bayesian model in the case of a heterogenous portfolio (consisting of very different assets, like bonds and equities) should be empirically checked.

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REFERENCES

[1] Bauwens L., Laurent S., Rombouts J.V.K., [2006], *Multivariate GARCH models: A survey*, „Journal of Applied Econometrics" 21, 79-109.

[2] Engle R., [2002], *Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models*, „Journal of Business and Economic Statistics" 20, 339–350.

[3] Jacquier E., Polson N., Rossi P., [1995], *Models and prior distributions for multivariate stochastic volatility*, technical report, University of Chicago, Graduate School of Business.

[4] O'Hagan A., [1994], *Bayesian Inference*, Edward Arnold, London.

[5] Osiewalski J., Pajor A., [2007], *Flexibility and parsimony in multivariate financial modelling: a hybrid bivariate DCC–SV model*, in: Milo W. and Wdowiński P. (ed.), *Financial Markets. Principles of Modeling, Forecasting and Decision-Making* (FindEcon Monograph Series No. 3), Łódź University Press, Łódź 2007 (11-26).

[6] Osiewalski J., Pajor A., Pipień M., [2007], *Bayesian comparison of bivariate GARCH, SV and hybrid models*, in: Welfe W. and Welfe A. (ed.), *MACROMODELS'2006, Proceedings of the 33rd International Conference*, Łódź 2007 (247-277).

[7] Osiewalski J., Pipień M., [2004], *Bayesian comparison of bivariate ARCH-type models for the main exchange rates in Poland*, „Journal of Econometrics" 123, 371-391.

[8] Pajor A., [2003], *Procesy zmienności stochastycznej SV w bayesowskiej analizie finansowych szeregów czasowych* (*Stochastic Volatility Processes in Bayesian Analysis of Financial Time Series*), doctoral dissertation (in Polish), published by Cracow University of Economics, Kraków.

[9] Pajor A., [2005a], *Bayesian analysis of stochastic volatility model and portfolio allocation*, Acta Universitatis Lodziensis – Folia Oeconomica, 192, 229-249.

[10] Pajor A., [2005b], *Bayesian comparison of bivariate SV models for two related time series*, Acta Universitatis Lodziensis – Folia Oeconomica 190, 177-196.

[11] Tierney L., [1994], *Markov chains for exploring posterior distributions* (with discussion), Annals of Statistics 22, 1701-1762.

[12] Tsay R.S., [2005], *Analysis of Financial Time Series* (2nd edition), Wiley, New York.

NOWE HYBRYDOWE MODELE WIELOWYMIAROWEJ ZMIENNOŚCI (PERSPEKTYWA BAYESOWSKA)

Streszczenie

W przypadku dużego portfela istniejące modele dynamicznej wielowymiarowej zmienności są albo zbyt proste z perspektywy finansowej, albo zbyt złożone z numerycznego punktu widzenia. W pracy proponuje się nową, hybrydową klasę modeli dla n-wymiarowych finansowych szeregów czasowych. Specyfikacje hybrydowe opierają się na dwóch prostych strukturach: modelu ze stochastycznym czynnikiem dyskontowym (stochastic discount factor, SDF) z klasy MSV i modelu skalarnym BEKK(1,1) z klasy MGARCH. W pracy definiuje się hybrydowe modele typu I i II; oba typy uwzględniają różną dynamikę każdej warunkowej wariancji czy kowariancji (jak w modelu BEKK) i zachowują tylko jeden proces ukryty w warunkowej macierzy kowariancji w celu opisu obserwacji odstających (jak w modelu SDF). Na potrzeby bayesowskich analiz *a posteriori* i predyktywnych zaproponowano podejście symulacyjne oparte na próbkowaniu Gibbsa oraz zasugerowano wersje przybliżone, nieuniknione w przypadku dużego n.

NEW HYBRID MODELS OF MULTIVARIATE VOLATILITY (A BAYESIAN PERSPECTIVE)

Summary

In the case of a large portfolio, the existing models of time-varying multivariate volatility are either too simple from the financial perspective or too complex from the numerical angle. Thus, in the paper a new hybrid class of models for n-variate financial time series is proposed. The hybrid specifications are based on two simple structures: the stochastic discount factor model (SDF) from the MSV class and the scalar BEKK(1,1) model from the MGARCH class. Type I and II hybrid models are defined; both allow for different dynamics of each conditional variance or covariance (like BEKK) and keep just one latent process in the conditional covariance matrix in order to describe outliers (like SDF). For the purpose of Bayesian posterior and predictive analyses, the simulation approach based on Gibbs sampling is proposed and approximations unavoidable in the case of large n are suggested.

Key words: Bayesian econometrics, Gibbs sampling, time-varying volatility, multivariate GARCH processes, multivariate SV processes.